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Course H 111 Verkeerskunde Basis

Traffic Flow Theory

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Preface

This part of the course Basics of Traffic Engineering (H111) deals with the theory of traffic flow. This theory studies the dynamic properties of traffic on road sections.

We begin this course with a theoretical framework in which the characteristics of traffic flow are described at the microscopic level. We then examine a number of dynamic models that were formulated on the basis of empirical research. We conclude with a discussion of some recent observations on congestion.

The theories and models that will be discussed are developed on the basis of numerous observations on motorways. There is a difference between motorways and lower order roads such as provincial roads and urban streets. For the latter it are the intersections that dominate flow characteristics to a large degree. Traffic flow on intersections is the subject of a separate workshop on Signal-Controlled Intersections (H112).

This text is a second version. Remarks and suggestions continue to be appreciated.

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Contents

1 Trajectories and microscopic variables

This chapter develops a theoretical framework in which the characteristics of traffic flows are described at the microscopic level. In a microscopic approach to traffic, each vehicle is examined separately.

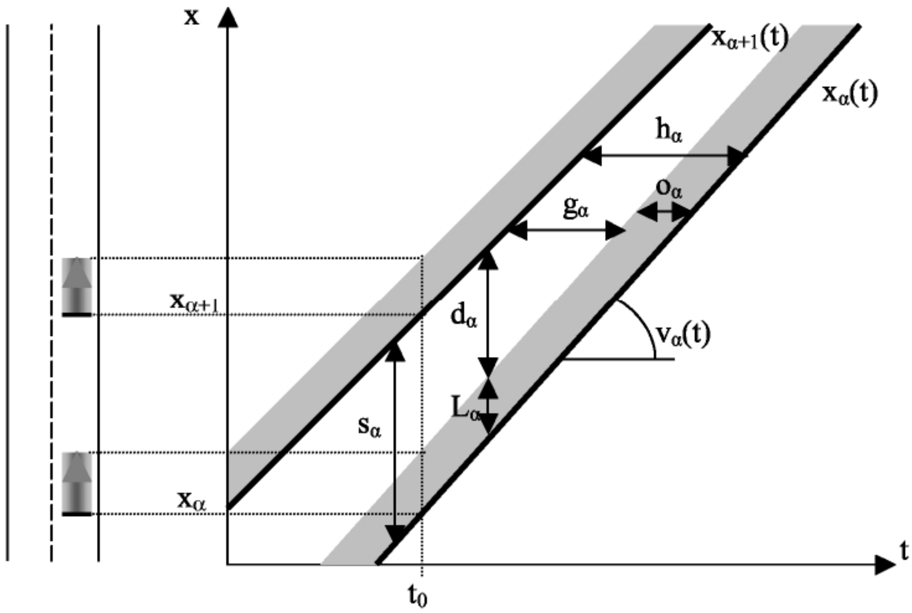

Figure 1 A road with two vehicles along an x-axis and the same vehicles in a t-x co-ordinate system

On the left side in Figure 1, along a vertical X-axis, x_a indicates the position of vehicle α at time t_0. The vehicle in front of this vehicle is indicated by α+1. Since both vehicles travel across the road, their positions are time dependent. The right side of Figure 1 presents the vehicles in a *t-x* co-ordinate system.

The position of a vehicle through time is called a trajectory. In this course we use the rear point, the rear bumper of a vehicle, as the point of reference for the trajectory of that vehicle. Figure 1 uses bold black lines to indicate the trajectories of vehicles α en α+1. The grey area represents the entire vehicle.

It is impossible for two trajectories to intersect when the vehicles travel on the same traffic lane. The speed v_a of a vehicle if given by the derivative with respect to the trajectory. The second derivative is the acceleration a_a. Accelerating cars have positive values for a_x and braking cars have negative values for a_a.

$$v_\alpha(t) = \frac{dx_\alpha(t)}{dt} \qquad (1.1)$$

$$a_\alpha(t) = \frac{d^2 x_\alpha(t)}{dt^2} \qquad (1.2)$$

A vehicle occupies a specific part of the road. This space occupancy or simply *space* s_a consists of the physical length of the vehicle L_a and the *distance* d_a kept by the driver to the vehicle in front, or:

$$s_\alpha(t) = x_{\alpha+1}(t) - x_\alpha(t) \qquad (1.3)$$

$$s_\alpha(t) = d_\alpha(t) + L_\alpha \qquad (1.4)$$

Analogously to space, vehicles also use a certain segment of time which is called *headway* h. This headway time consists of the interval time or *gap* g and the *occupancy* o.

$$h_\alpha = g_\alpha + o_\alpha \qquad (1.5)$$

At constant speeds, or in general when acceleration is neglected, occupancy becomes:

$$o_\alpha = \frac{L_\alpha}{v_\alpha} \qquad (1.6)$$

The speed difference Δv is given by:

$$\Delta v_\alpha(t) = v_{\alpha+1}(t) - v_\alpha(t) = \frac{ds_\alpha(t)}{dt} \qquad (1.7)$$

These variables can all be measured. Two aerial photographs taken in quick succession give us the positions, the speeds, the occupancies, the headways and the gaps. Using detection loops (that work on the magnetic-induction-principle) and detection cameras the speed, space, length and distance of vehicles can be measured fairly inexpensively.

Roads usually show a variety of vehicle types and drivers. We call the idealised traffic state with only one type of road user *homogeneous*. A traffic state is *stationary* when it does not change over time. When this is the case, vehicles on homogeneous roads share the same speeds and trajectories are straight lines.

2 Macroscopic variables

At the macroscopic level we do not look at the vehicles as separate entities. The traditional traffic demand model discussed elsewhere in this course is a macroscopic model. This macroscopic level is also relevant to the dynamic description of traffic. This section defines the macroscopic variables that translate the discrete nature of traffic into continuous variables.

2.1 A measurement interval

A measurement interval *S* is defined as an area in the *t*-*x* space. When macroscopic variables are defined later on, it is always done for a certain measurement interval. Figure 2 and Figure 3 below show some measurement intervals:

- S_1: This rectangular measurement interval covers a road section of length ΔX during an infinitely small time interval *dt*. This coincides approximately with a location interval ΔX at a specific moment t_1. We assume that *n* vehicles move through this interval and in the text we shall indicate them by index *i*. Such a location interval could be recorded from an aeroplane on an aerial photograph.
- S_2: This rectangular measurement interval represents an infinitely small road length *dx* during a time interval of ΔT. This coincides approximately with a time interval ΔT at a location x_2. In further derivations we assume that *m* trajectories cross this measurement interval and for these *m* vehicles we use the index *j*. Induction loops and detection cameras have been placed on several locations of our road network and these measure the traffic during time intervals.

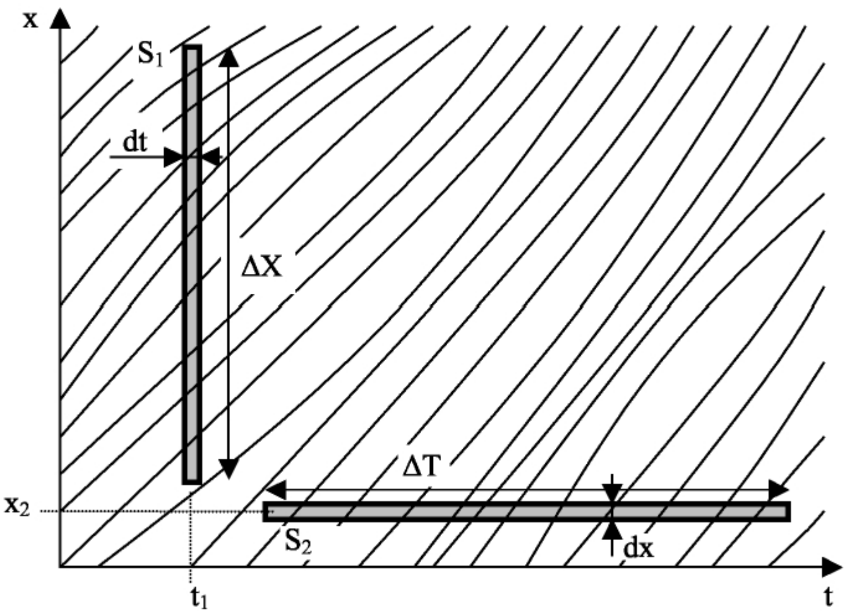

Figure 2 Trajectories and the measurement intervals S_1 and S_2

- S_3 is an arbitrary measurement interval in time and space. This measurement interval has an area Opp(S_3) with dimensions time * space . Several trajectories traverse this measurement interval. The distance travelled by a vehicle in the measurement interval is the projection of its trajectory on the *x*-axis. The time spent by this vehicle in the measurement interval is the projection of the matching trajectory on the time-axis.

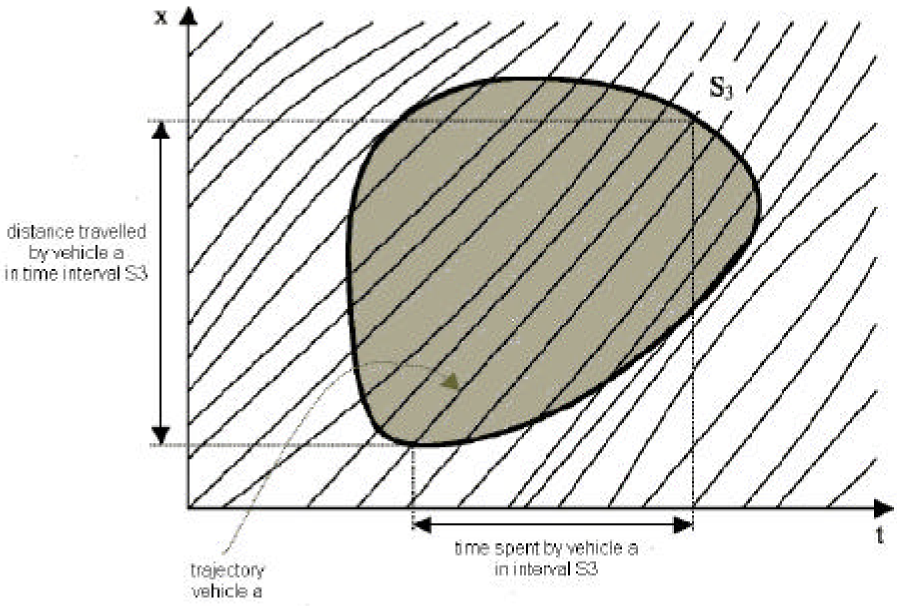

Figure 3 The measurement interval S_3

2.2 Density

Density is a typical variable from physics that was adopted by traffic science. Density *k* reflects the number of vehicles per kilometre of road. For a measurement interval at a certain point in time, such as S_1, *k* can be calculated over a road section with ΔX length as:

$$k = \frac{n}{\Delta X} \qquad (2.1)$$

The index *n* indicates the number of vehicles at t_1 on the location interval ΔX. Total space of the *n* vehicles can be set equal to ΔX, thus:

$$k = \frac{n}{\sum_n s_i} = \frac{1}{\bar{s}} \qquad (2.2)$$

Figure 4 Location interval S_1

where the mean space occupancy in the interval S_1 is defined as:

$$\bar{s} = \frac{1}{n}\sum_{n} s_i \qquad (2.3)$$

Density k depends on the location, time and the measurement interval. We will, therefore, rewrite formula (2.1), in order to include these dependent factors in our notation. For the location x_1 we take the centre of the measurement interval ΔX.

$$k(x_1, t_1, S_1) = \frac{n}{\Delta X} \qquad (2.4)$$

Density is traditionally expressed in vehicles per kilometre. Maximal density on a road fluctuates around 100 vehicles per kilometre per traffic lane.

The density definition in (2.4) is confined to a certain point in time. The next step is to generalise this definition If we multiply numerator and denominator of (2.4) by the infinitely small time interval dt around t_1, density becomes:

$$k(x_1, t_1, S_1) = \frac{n.dt}{\Delta X.dt} \qquad (2.5)$$

The denominator of (2.5) now becomes equal to the area of the measurement interval S_1. The numerator reflects the total time spent by all vehicles in the measurement interval S_1.

$$k(x_1, t_1, S_1) = \frac{\text{Total time spent in S1}}{\text{Area (S1)}} \qquad (2.6)$$

In the same way we define the density at location x, at time t and for a measurement interval S as:

$$k(x, t, S) = \frac{\text{Total time spent by all vehicles in S}}{\text{Area (S)}} \qquad (2.7)$$

By way of illustration:
Density according to definition (2.7) for x_2, t_2 in the measurement interval S_2, as illustrated once more in Figure 5:

$$k(x_2, t_2, S_2) = \frac{\sum_{m} \frac{dx}{v_j}}{\Delta T.dx} = \frac{\sum_{m} \frac{1}{v_j}}{\Delta T} \qquad (2.8)$$

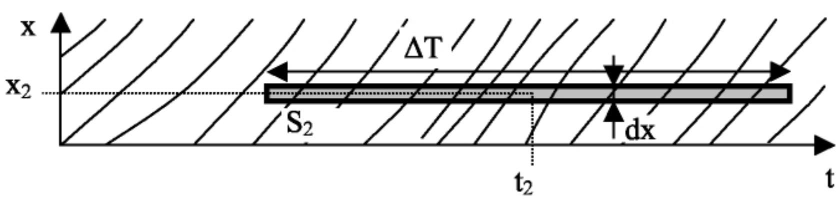

Figure 5 Time interval *S*₂

2.3 Flow rate

The flow rate q can be compared to the discharge or the flux of a stream. The flow rate represents the number of vehicles that passes a certain cross-section per time unit. For a time interval ΔT at any location x_2, such as the measurement interval S_2 in Figure 5, the flow rate is calculated as follows:

$$q(x_2,t_2,S_2)=\frac{m}{\Delta T} \qquad (2.9)$$

The index m represents the number of vehicles that passes location x_2 during ΔT. This time interval is the sum of the m headways. Through the introduction of a mean headway $\bar{h}$ we find the following expression for the traffic flow rate:

$$q=\frac{m}{\sum_m h_j}=\frac{1}{\bar{h}} \qquad (2.10)$$

The flow rate is expressed in vehicles per hour. We call the maximum possible flow rate of any road its capacity. Depending on vehicle composition, the capacity of a motorway lies between 1800 and 2400 vehicles per hour per traffic lane.

This definition of flow rate (2.9) is limited to a time interval. We get a more general definition by multiplying the numerator and the denominator with an infinitely small location interval dx around x_2. The denominator again becomes the area of the measurement interval and the numerator equals the total distance travelled by all vehicles in the measurement interval.

$$q(x_2,t_2,S_2)=\frac{m.dx}{\Delta T.dx}=\frac{\text{Total distance covered by vehicles in } S_2}{\text{Area } (S_2)} \qquad (2.11)$$

This leads to a general definition for flow rate:

$$\boxed{q(x,t,S)=\frac{\text{Total distance covered by vehicles in S}}{\text{Area (S)}} \qquad (2.12)}$$

By way of illustration:

Applying (2.12) we calculate the flow rate for the measurement interval S_1, at location x_1 and time t_1:

$$q(x_1,t_1,S_1)=\frac{\sum_n v_i.dt}{dt.\Delta X}=\frac{\sum_n v_i}{\Delta X} \qquad (2.13)$$

2.4 Mean speed

We define the mean speed u as the quotient of the flow rate and the density.
The mean speed is also a function of location, time and measurement interval. Note that

the area of the measurement interval no longer appears in definition (2.14):

$$u(x,t,S) = \frac{q(x,t,S)}{k(x,t,S)} = \frac{\text{Total distanc covered by vehicle s in S}}{\text{Total time spent by vehicle s in S}} \qquad (2.14)$$

In another form this definition of the mean speed is also called the fundamental relation of traffic flow theory:

$$q = k.u \qquad (2.15)$$

This relation irrevocably links flow rate, density and mean speed. Knowing two of these variables immediately leads to the remaining third variable.

We calculate the mean speed for the measurement intervals S_1 and S_2 as follows:

For the location interval S_1 the density is given by (2.5) and the flow rate by (2.13). The mean speed for these *n* vehicles in the interval S_1 at location x_1 and point in time t_1 then becomes:

$$u(x_1,t_1,S_1) = \frac{1}{n}\sum_n v_i \qquad (2.16)$$

We get the mean speed for a location interval by averaging the speeds of all of the vehicles in this interval.

For the time interval S_2, density was calculated in (2.8) and flow rate in (2.9). The mean speed for *m* vehicles then becomes:

$$u(x_2,t_2,S_2) = \frac{1}{\frac{1}{m}\sum_m \frac{1}{v_j}} \qquad (2.17)$$

This shows that the mean speed over a time interval is the *harmonic* mean of the individual speeds.

If we take the normal arithmetical average of the individual vehicle speeds in a time interval we get the *time-mean speed* u_t, as defined in (2.18):

$$u_t(x_2,t_2,S_2) = \frac{1}{m}\sum_m v_j \qquad (2.18)$$

This time-mean speed u_t differs from the mean speed u and does therefore, NOT comply with the fundamental relation (2.15).

The difference between the mean speed and the time-mean speed is illustrated by the example below:

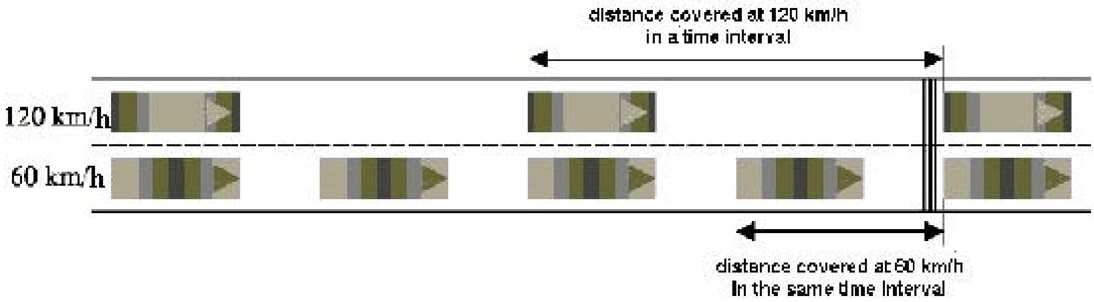

Figure 6 Motorway with two traffic lanes.

Consider a long road with two traffic lanes, where all vehicles on the right traffic lane travel at 60 km/h and the vehicles on the left lane at 120 km/h. All the vehicles on the first traffic lane that passed a detector during a 1 minute time interval can be found on a 1 kilometre long road section. For the left traffic lane, the length of this road section equals 2 kilometres. Thus, when the time-mean speed is assessed, faster cars are considered over a much longer road section than slower cars. When we calculate mean speed, and also when we calculate density, the length of the road section used is the same for fast and slow cars. Therefore, the proportion of fast vehicles is overestimated when calculating time-mean speed thus making it always larger than or equal to the mean speed.

Example problem:

Assume for that 1200 vehicles/hour pass on both traffic lanes in the example above. What are the density, the flow rate, the mean speed and the time-mean speed on this road?

Solution:

q = *2400 vehicles/hour*
k = *30 vehicles/km*
u = *80 km/hour*
u_t = *90 km/hour*

Analogously we can also define the space-mean speed u_x for a location interval as the mean of the speeds of all vehicles in this location interval or:

$$u(x_1, t_1, S_1) = \frac{1}{n}\sum_{n} v_i \qquad (2.19)$$

Equation (2.16) shows that the space mean speed equals the mean speed as defined in (2.14).

Thus we distinguish three definitions: the mean speed u, the time-mean speed u_t and the space-mean speed u_x. Here u always equals u_x and the fundamental relation applies to these definitions. The time-mean speed u_t is different and does NOT comply with the fundamental relation.

2.5 Relative occupancy

Most traffic measurements are carried out at a fixed location x_2. The occupancy o of a vehicle is easy to measure in such cases. The relative occupancy b in time interval S_2 is given by:

$$b(x_2, t_2, S_2) = \frac{1}{\Delta T} \sum_m o_j \qquad (2.20)$$

If we assume that all vehicles have the same length, we get a relation between the relative occupancy b and the density k. Verify that inserting (1.6), (2.9), (2.17) and (2.15) in (2.20) leads to:

$$b(x_1, t_1, S_1) = L.k(x_1, t_1, S_1) \qquad (2.21)$$

Example:

Consider a stream of traffic with mean speed of 60 km/h and a flow rate of 1200 vehicles/hour. All vehicles are 4 meters in length. What is the relative occupancy?

The density $k = q / u$ = 20 vehicles/km.
A density of 20 vehicles/km means a space occupancy of 50 metres per vehicle. The vehicle takes 4 metres, or 8% of the space. So relative occupancy = 8%.
Computing the density and length in (2.21) also gives:
$b = L \,.\, k = 0.004 \,.20 = 8\ \%$

This formula cannot be used in practical situations because a traffic stream is never homogeneous in reality. If we want to find traffic density by using traffic detectors, it is better to measure the flow rate and mean speed using (2.8) and (2.17) and then to calculate density using the fundamental relation (2.15).

2.6 Conclusion

The macroscopic traffic variables can be calculated for every location, at any point in time and for every measurement interval. In practice we mostly use traffic detectors that measure the macroscopic variables u and q across a certain time interval. If we want to calculate the mean speed u for a time interval, we must take the harmonic mean of the individual speeds. The discrete nature of traffic requires time intervals of at least half a minute if we want to achieve meaningful information. When the time intervals exceed a duration of five minutes, certain dynamic characteristics are lost.

3 Fundamental diagram

The previous chapter defined three macroscopic variables: flow rate q, density k and mean speed u. Because of the fundamental relation $q = k.u$ (2.15) there are only two independent variables. This chapter introduces an empirical relation between the two remaining independent variables. We do this by assuming stationary (flow rates do not change along the road and over time) and homogeneously composed traffic flow (all vehicles are equal). This means that we can simplify the notation somewhat because the dependence on location, time and measurement interval no longer applies in a stationary flow.

3.1 Observations.

On a three-lane motorway we measured the flow rate q and the mean speed u during time intervals of one minute. Each observation, therefore, gives a value for the mean speed u and a value for the flow rate q. Figure 7 shows the different observation points in a q-u diagram.

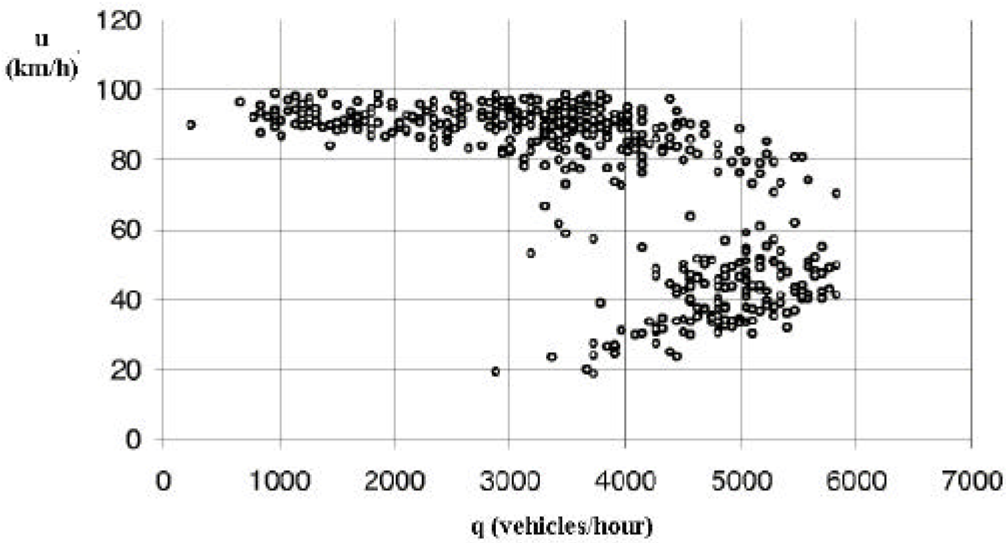

Figure 7 Observation points in a *q-u* diagram

We calculate the density k (= q / u) for each observation. This means that the points of observation can also be plotted in a k-q diagram (Figure 8) and a k-u diagram (Figure 9).

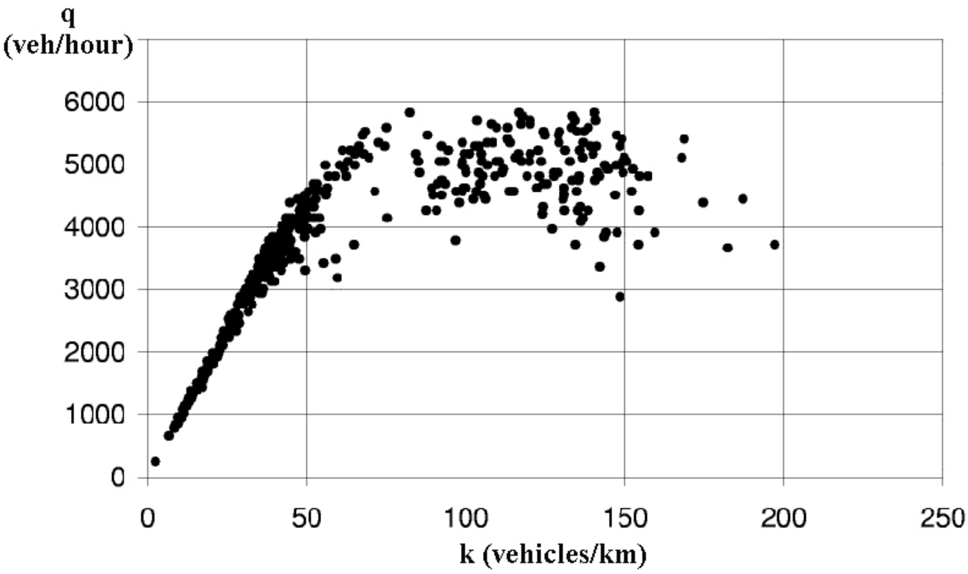

Figure 8 Observation points in a *k-q* diagram.

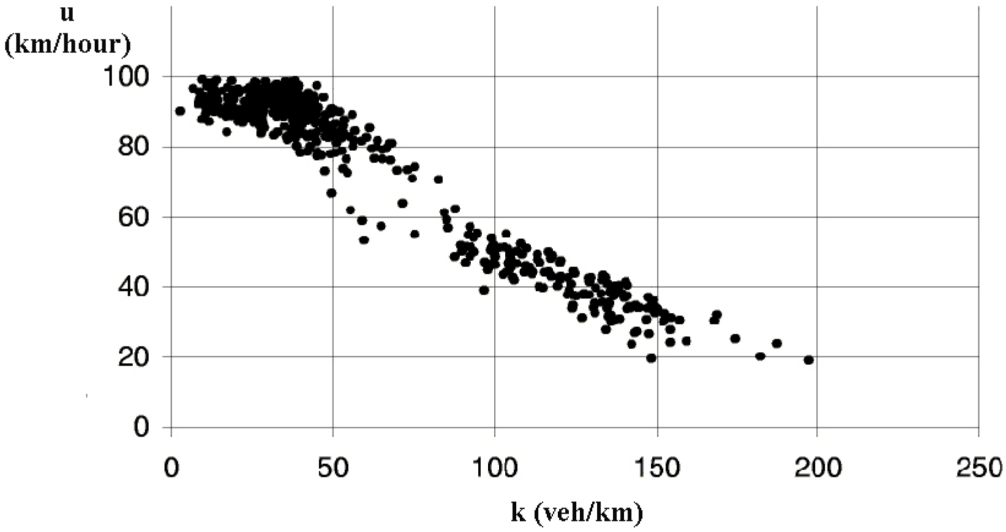

Figure 9 Observation points in a *k-u* diagram.

The observations were carried out on an actual motorway where traffic is not homogeneous: there is a variety of vehicle types and drivers behave in a variety of ways. Nor is real traffic stationary: vehicles accelerate and decelerate continuously. Abstracting from the inhomogeneous and non-stationary characteristics, we can describe the empirical characteristics of traffic using an equilibrium relation that we can present in the form of the three diagrams shown above.

3.2 The fundamental diagrams

Road traffic is always in a specific state that is characterised by the flow rate, the density and the mean speed. We combine all the possible homogeneous and stationary traffic states in an equilibrium function that can be described graphically by three diagrams. The equilibrium relations presented in this way are better known under the name of fundamental diagrams. Figure 10 sketches them and it shows the relationship between each of the diagrams..

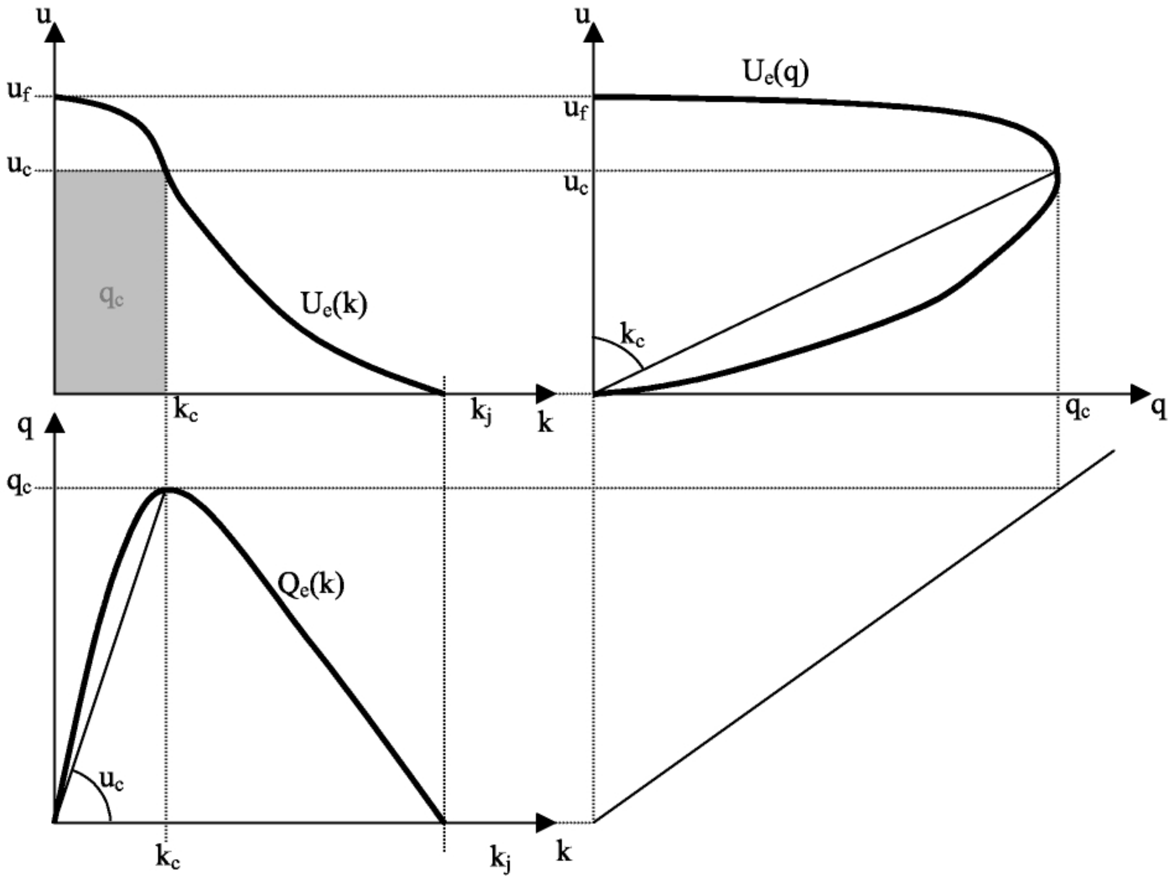

Figure 10 The three related fundamental diagrams

A diagram shows the relation between two of the three variables. The third variable can always be recovered by means of the relationship $q = k.u$. The third variable in the *q-u* and the *k-q* diagram is an angle. The flow rate in the *k-u* diagram is represented by an area. A fundamental diagram applies to a specific road and is drawn up on the basis of observations. Thus stationary and homogeneous traffic is always in a state that is located on the bold black line. Some special state points require extra attention:

- *Completely free flowing traffic*
  When vehicles are not impeded by other traffic they travel at a maximum speed of u_f (*free speed*). This speed is dependent, amongst other things, on the design speed of a

road, the speed restrictions in operation at any particular time and the weather. At free speed, flow rate and density will be close to zero.

- *Saturated traffic*
  On saturated roads flow rate and speed are down to zero. The vehicles are queuing and there is a maximum density of k_j (*jam density*).
- *Capacity traffic*
  The capacity of a road is equal to the maximum flow rate q_c. The maximum flow rate of q_c has an associated capacity speed of u_c and a capacity density of k_c. The diagram shows that the capacity speed u_c lies below the maximum speed u_f .

3.3 Mathematical models for the fundamental diagrams

In this section we present mathematical expressions for the equilibrium relations given by the fundamental diagrams. We examine the original diagram of Greenshield and the triangular diagram.

- Greenshield (1934)

Greenshield drew up a first formulation that was based on a small number of slightly questionable measurements. In this formulation the relation in the *k-u* diagram is assumed to be linear, leading to parabolic relations for the remaining diagrams (see **Error! Reference source not found.**).

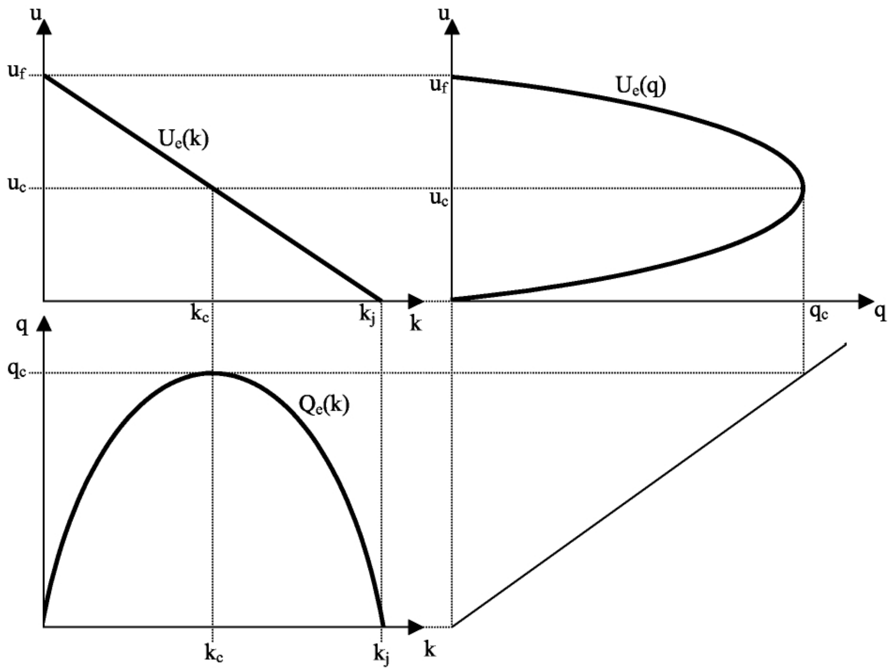

Figure 11 The fundamental diagrams according to Greenshield

In Greenshield's diagrams, the capacity speed u_c is half the maximum speed u_f. The capacity density k_c in this model is half the maximum density k_j. This formulation is a rough simplification of observed traffic behaviour, but is still frequently used because of its simplicity and for historical reasons. The equilibrium function in the *k-u* diagram can be written as::

$$u = U_e(k) = \frac{u_f}{k_j}(k_j - k) \qquad (3.1)$$

Applying the fundamental relation gives the other relations ($Q_e(k)$ and $U_e(q)$). Note that the relation $U_e(q)$ is not a function!

$$q = Q_e(k) = \frac{u_f}{k_j} k(k_j - k) \qquad (3.2)$$

$$q = U_e(q)^{-1} = k_j u(1 - \frac{u}{u_f}) \qquad (3.3)$$

- Triangular diagram

A second much-used formulation assumes that the fundamental *k-q* diagram is triangular in shape. This simple diagram has many advantages in dynamic traffic modelling, as will be discussed in Chapter 4.

In this equilibrium relation the mean speed equals the maximum speed for all traffic states that have densities smaller than the capacity density. The branch of the triangle that links the capacity state with the saturated state, has a negative constant slope *w*. Figure 12 represents this triangular diagram.

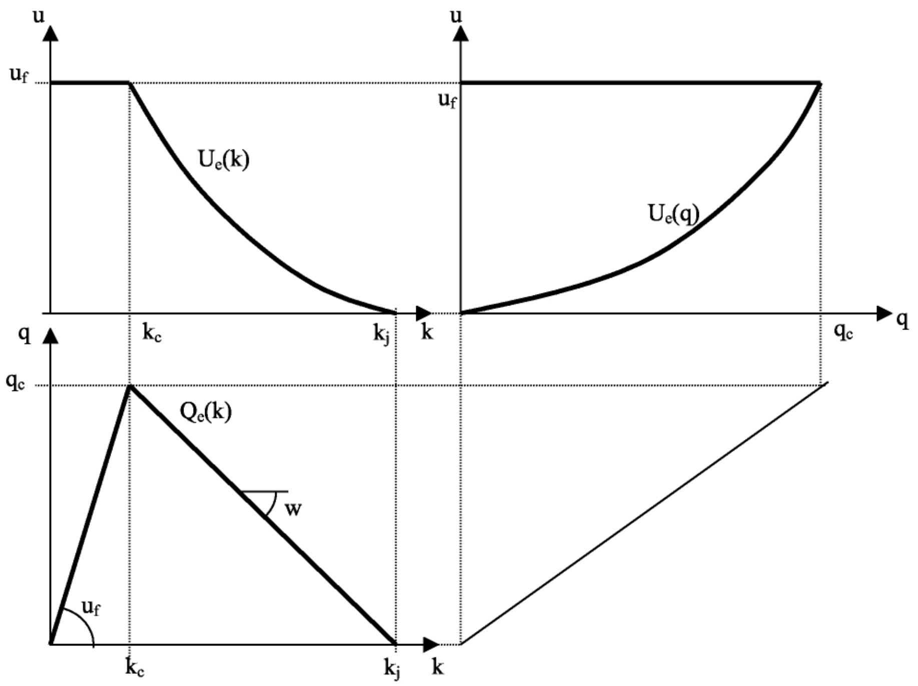

Figure 12 The fundamental diagrams when a triangular *k-q* diagram applies

4 Macroscopic traffic flow model

In the two previous chapters we learned that the fundamental relation (*q=k.u)* and the fundamental diagrams (Figure 10) enable us to describe the traffic state of stationary and homogeneous traffic. Thus we can calculate the two remaining variables for a given value of a macroscopic variable. When traffic is stationary and homogeneous, we know that the values for these variables will remain constant along the entire road and for some extended period.
However, real traffic is neither homogeneous nor stationary. In this chapter our aim is to describe the evolution of traffic over time. In doing so, we will ignore the dependency on the measurement interval *S* in the notation in order to discover the dynamic relation between *q(x,t), u(x,t)* and *k(x,t).* We assume, therefore, that we are dealing with point variables: variables that are singularly defined at any moment and at every location. By doing this we can show these three variables as functions in the *t-x* plane.

4.1 Derivation and formulation

We use a traffic conservation law to describe the changes in time and location of the macroscopic variables along a road. The fundamental relation *q(x,t)=k(x,t).u(x,t)* continues to apply.

We divide the road to be modelled in cells with a length of Δx. The density of cell *i* at time t_j is indicated by *k(i,j).* The number of vehicles in the cell is *k(i,j).*Δx . One time interval Δt later, at t_j+1, density has changed as follows (see Figure 14):

- A number of vehicles travelled from cell *i*-1 into cell *i.* The expected inflow is given by *q(i*-1,*j*). Δt
- From cell *i* a number of vehicles travelled to cell *i*+l. This outflow is given by *q(i,j).* Δt
- Feeder- and exit roads enable in- and outflows that are indicated by *z(i,j).*Δx.Δt where *z* is expressed per time- and length-unit and is taken positive for an increase in the number of vehicles.

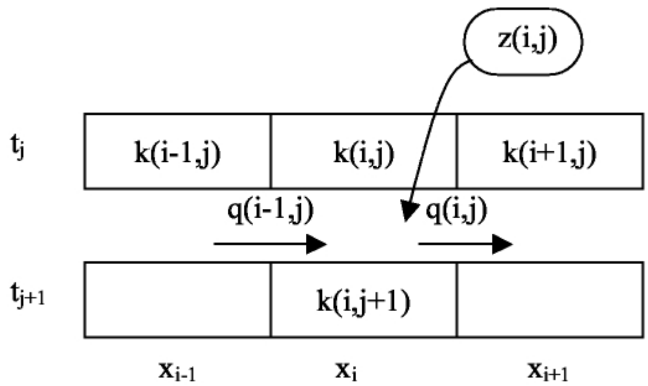

Figure 13 Derivation of the conservation law

For cell i at time t_j we can now write the following formulation of the state:

$$k(i, j+1).\Delta x = k(i, j).\Delta x + q(i-1, j).\Delta t - q(i, j).\Delta t + z(i, j).\Delta x.\Delta t \qquad \textbf{(4.1)}$$

or:

$$\frac{k(i, j+1) - k(i, j)}{\Delta t} + \frac{q(i, j) - q(i-1, j)}{\Delta x} = z(i, j) \qquad \textbf{(4.2)}$$

Taking the limit with respect to the time step and letting cell length approach zero results in the following partial differential equation representing the *conservation law of traffic*:

$$\frac{\partial k(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = z(x,t) \qquad \textbf{(4.3)}$$

We add another assumption to this conservation law: All possible dynamic traffic states comply with the stationary fundamental diagrams. This means that although traffic states on roads can change over time, they still comply with the fundamental diagrams at each moment and at every location. Therefore the successive traffic states 'move' as it were across the bold black lines in the fundamental diagrams.

This assumption allows us to write the flow rate in function of density as follows:

$$q(x,t) = Q_e(k(x,t)) \qquad \textbf{(4.4)}$$

Inserting (4.4) in (4.3) and applying the chain rule gives a partial differential equation that only contains partial derivatives with respect to density.

$$\frac{\partial k(x,t)}{\partial t} + \frac{dQ_e(k(x,t))}{dk}\frac{\partial k(x,t)}{\partial x} = z(x,t) \qquad \textbf{(4.5)}$$

In (4.5) the expression $z(x,t)$ represents the volume of traffic that enters the road at time t and location x (a negative value for exiting traffic) and $dQe(k)/dk$, or in short $Q_e'(k)$ represents the derivative of the fundamental k-q function. In the subsequent derivation we assume a concave fundamental diagram which means that $Q_e'(k)$ will always decrease for increasing densities.

Using the fundamental diagram in the traffic conservation law led to the first dynamic traffic model in the 1950s. This model was named after the people who first proposed it: the LWR-model (Lighthill, Whitham, Richards). Several schemes were developed to numerically solve this equation with the help of a computer in order to obtain a traffic model that could be applied to practical situations. In the following section we will study this equation in an analytical way in order to gain some insight into some of the dynamic characteristics of a traffic stream.

4.2 Characteristics

The partial differential equation (4.5) is known in mathematical analysis as the "Burgers equation". It can be solved analytically with the help of given boundary conditions. If we apply the equation to a road without feeder- and exit lanes and if, for the sake of convenience, we assume that $Q_e'(k)$ equals c, the conservation equation (4.5) can be simplified to:

$$\frac{\partial k(x,t)}{\partial t} + c\frac{\partial k(x,t)}{\partial x} = 0 \qquad (4.6)$$

Solving this equation means finding the traffic density on this road in function of time and location. The solution to this equation is given by:

$$k(x,t) = F(x-ct). \qquad (4.7)$$

where F is an arbitrary function. By inserting (4.7) in (4.6) we can verify that (4.7) does indeed solve the partial differential equation. The solution implies that when x-ct is constant, density also remains constant. This means that all points on a straight line with slope c have the same density.

Example:

> For a point on the x-axis ($x = x_0$ and $t = 0$) equation (4.7) gives: $k(x_0,0) = F(x_0)$.
> At (x_0+ct, t) density $k(x_0+ct, t)$ also equals $F(x_0)$.
> Thus all points on the straight line with slope c through $(x_0,0)$ have the same density equal to $k(x_0,0)$.

If we know the value of the density at a point, we can draw a straight line through that point with slope c. The density then remains constant along this line. Such a straight line is known as a *solution line* or *characteristic*.

We sketch the t-x diagram in Figure 14a. Assume that the initial value in x_0 equals k_0. A straight line with slope c can then be drawn through x_0 along which density also equals k_0.

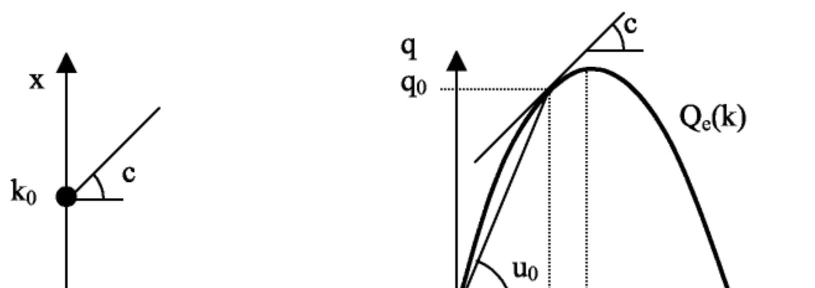

Figure 14 (a) the *t*-*x* diagram and (b) the *k*-*q* fundamental diagram

In actual fact the value of c equals $Q_e'(k_0)$. This is the derivative of the fundamental diagram function for k_0. In other words, c equals the slope of the tangent to the fundamental k-q diagram in k_0. We can now draw the k-q diagram to scale with the t-x

diagram so that equal slopes in both diagrams correspond to the same speed. It is now possible to draw a line parallel to the tangent to the fundamental diagram through a point in the *t*-*x* diagram where we know the initial condition.

From the initial- and boundary conditions we can draw solution lines where the traffic state is known. A specific value for the density k_0 always corresponds to an associated flow rate q_0 and a mean speed u_0. Along a characteristic both density, flow rate and mean speed remain constant. Note, from the fundamental diagram in Figure 14b, that vehicle speed always exceeds the speed *c* of the characteristics.

We divide the various traffic states into traffic regimes according to the slope of the characteristics:

- *Free flow*
  When density lies below the capacity density k_c, we speak of free flow. During this regime the mean speed of the traffic stream exceeds the capacity speed u_c . During free flow the speed of the characteristics $c = Q_e'(k)$ is positive. As a result, the characteristics run in the same direction as the traffic flow. This means that the properties of the traffic flow propagate in the same direction as the traffic flow itself (see Figure 14). The slope of the characteristics *c*, however, is always below the mean vehicle-speed u_0. Thus the properties of the traffic regime move at a lower speed than the individual vehicles.

- *Congested flow*
  When traffic speed lies below the capacity speed u_c or when traffic density lies between the capacity density k_c and the maximum density k_j we speak of congested flow. It is the regime in which tailbacks develop. During congestion $Q_e'(k)$ is negative. The characteristics run opposite to the direction of travel (see Figure 15) and the properties of the traffic flow propagate against the direction of the vehicle stream.

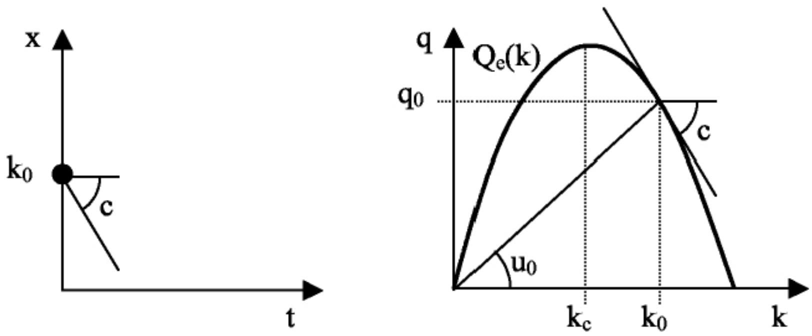

Figure 15 (a) the *t*-*x* diagram and (b) the fundamental diagram at congested flow

- *Capacity flow*
  Capacity flow is considered to be a separate regime. In this regime the flow rate is maximal. At capacity flow $Q_e'(k)$ equals zero and the characteristics run parallel to

the time-axis. This regime cannot propagate in either direction relative to the traffic stream. Capacity flow remains at the same location and functions as an upstream boundary for congested flow and a downstream boundary for free flow. We call the locations where this traffic regime occurs the bottlenecks in a traffic network.

The table below gives an summary:

Traffic regime	k	c	Direction of characteristics
Free flow	$k < k_c$	$c = Q_c'(k) > 0$	With traffic stream
Capacity flow	$k = k_c$	$c = Q_c'(k) = 0$	Stationary
Congested flow	$k > k_c$	$c = Q_c'(k) < 0$	Against traffic stream

Example:

In Figure 16 the characteristics (bold lines) and trajectories (dotted lines) are drawn when the initial- and boundary conditions are known: everywhere a density of k_0. Note that we could, in fact, draw an infinite number of characteristics. The number of trajectories, on the other hand, is finite. A characteristic is, by definition, a straight line along which density is constant. In this example the density is constant along all curves, including the trajectories.

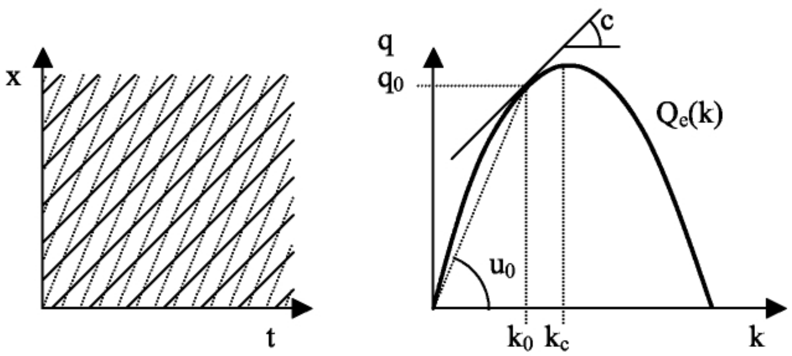

Figure 16 Trajectories and characteristics at homogeneous initial- and boundary conditions.

4.3 Shock waves

Consider a road with two traffic densities at time $t = 0$. For $x < x_0$ traffic density is k_1 and for $x > x_0$ density is k_2 with $k_2 > k_1$.

We call the transition between the two traffic states in x_0 a *front*. Characteristics upstream of the front depart with a speed of $c_1 = Q_e'(k_1)$. Downstream of x_0 characteristics depart with a speed of $c_2 = Q_e'(k_2)$. Since k_1 is smaller than k_2 and since the fundamental diagram is concave, the speed of the characteristics upstream of the front will exceed c_2. Thus is would appear that the different characteristics cross each other (see Figure 17). This, however, is impossible: on a well-specified location in the t-x diagram only one traffic state can prevail. Thus there must be a clear transition between these two traffic states. At time $t = 0$ this transition occurs at location x_0. We will see how this front moves over time.

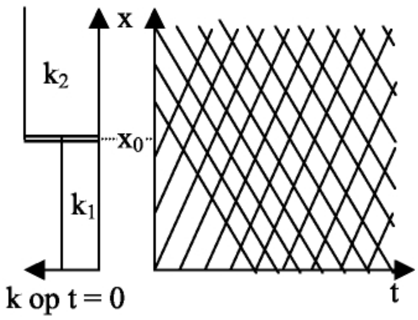

Figure 17 Characteristics at an increase in density

Let us assume that this front travels at speed U_{12} and consider the traffic flow across the front.

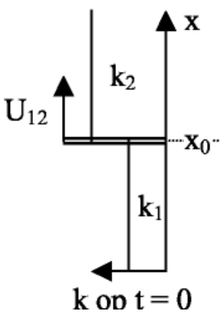

Figure 18 Calculating the speed of the front

The flow rate upstream of the front is $q_1 = k_1 . u_1$. A moving observer sees a relative flow rate that depends on his own travel speed. An observer with speed U_{12} just upstream of the front sees a relative flow rate $q_{r1}=k_1.(u_1 - U_{12})$. An observer with the same speed of U_{12} just downstream from the front sees a relative intensity of $q_{r2} = k_2.(u_2 - U_{12})$. If we assume that our observer moves with the front, he sees a relative flow rate q_{r1} upstream of the front and downstream he sees a relative flow rate q_{r2}. Since the conservation of vehicles also applies to the front, these two relative flow rates are equal or:
$q_{r1} = k_1.(u_1 - U_{12}) = q_{r2} = k_2 (u_2 - U_{12})$

This gives us the speed of the front as being:

$$U_{12} = (q_1 - q_2) / (k_1 - k_2) \qquad (4.8)$$

Thus the tailback front from the initial condition travels at a speed of U_{12} in the form of a shock wave. Characteristics end in the shock wave and the traffic state shows a discontinuous change. Trajectories that cross a shock wave change their speed abruptly.

The speed of the shock wave can also be read graphically from the fundamental diagram. To this end we indicate the two traffic states by the co-ordinates (k_1,q_1) and (k_2,q_2) as shown in Figure 19b. The slope of the connecting line between these two points is U_{12}. Consequently, the shock wave in the *t-x* diagram runs parallel to the connecting line between the two traffic states in the fundamental diagram. In this way we can draw the shock wave graphically in the *t-x* diagram, see Figure 19a.

We will now look at the direction in which the shock wave propagates. Since the density of the downstream traffic exceeds that of the traffic upstream, the sign of U_{12} equals the sign of ($q_2 - q_1$). When the downstream flow rate exceeds the upstream flow rate, as in Figure 19, the shock wave moves in the traffic stream direction. When the downstream flow rate is smaller than the upstream flow rate, the shock wave moves against the traffic stream.

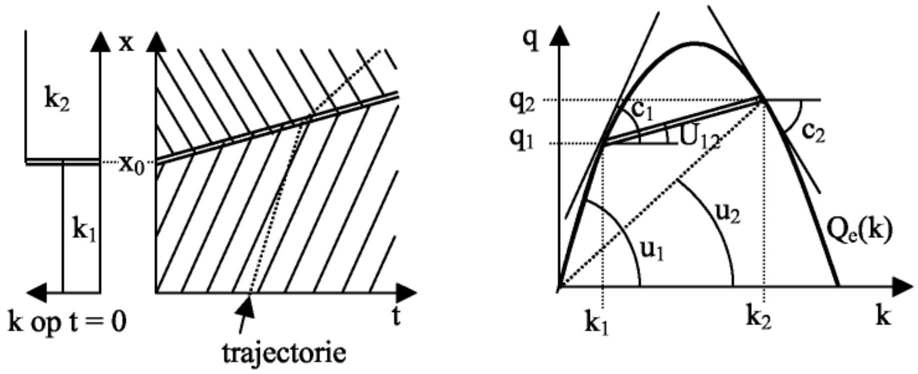

Figure 19 A shock wave in (a) the *t-x* diagram and (b) in the fundamental *k-q* diagram

Example:
We examine the evolution of traffic on a road (Figure 20) with traffic state 'A' as the initial condition for all points $x < x_1$ and for all points $t > 0$ on the boundary $x = 0$. In addition the initial condition 'B' applies between x_1 and x_2, and downstream for $x > x_2$, the traffic state is 'C'.

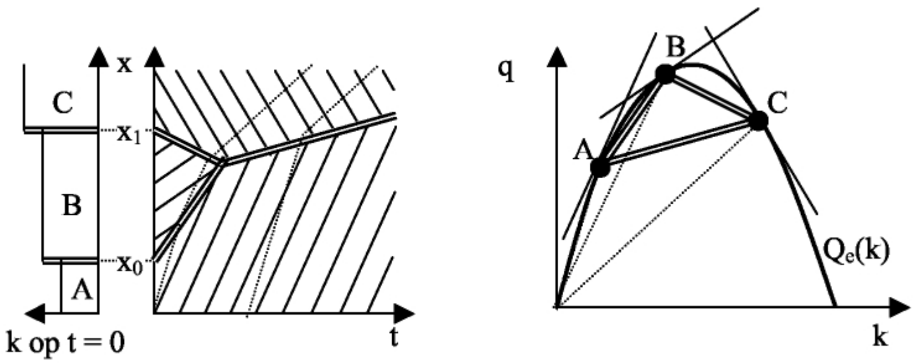

Figure 20 Merging shock waves

Starting from on the initial- and boundary conditions we can draw characteristics that run parallel to the tangents of the associate traffic states in the fundamental diagram. The shock wave between traffic states 'A' and 'B' runs with the direction of flow. The shock wave between 'B' and 'C' runs against the traffic flow. Traffic state 'B' disappears at the point where these two shock waves meet. At this point a shock wave starts that separates traffic states 'A' and 'C'.

4.4 Fans

We just saw that shock waves emerge when the density downstream exceeds the upstream density. In Figure 21 we consider a road where the downstream density (k_1) is below the density upstream (k_2).

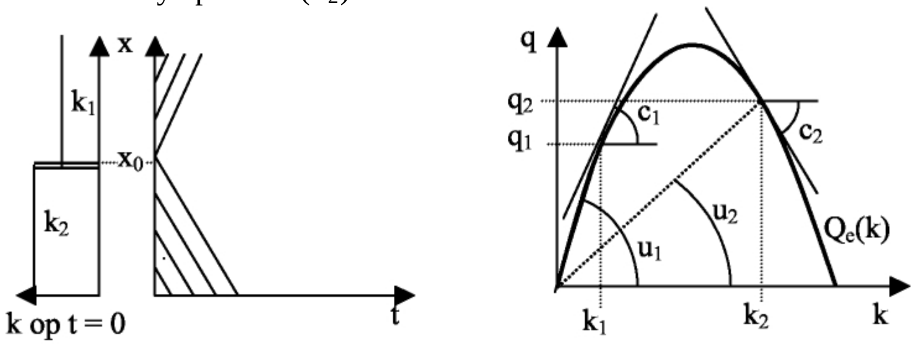

Figure 21 Characteristics at decreasing density

The characteristics downstream of x_0 show a speed c_1 that exceeds the downstream characteristic speed of c_2. This causes an empty space, as it were, in the *t-x* diagram between the characteristics that depart from x_0 at speeds c_1 and c_2. Since each point in the *t-x* diagram has a traffic state, we must find a solution to this problem.

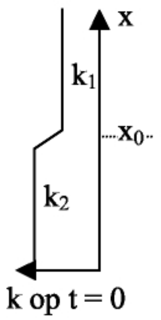

Figure 22 Spreading out an abrupt density change

Let us assume the abrupt transition of the traffic state k_2 to k_1 at x_0 to be a gradual one, as in Figure 22. In that case all intermediate densities occur and characteristics depart with all speeds possible between c_2 and c_1. A fan of characteristics, therefore, departs from x_0 with the result that all intermediate densities occur in the solution in the *t-x* diagram (Figure 23).

The horizontal characteristic in the fan corresponds to the capacity regime. This is an essential property of the LWR model: the transition from upstream congestion to a free flow traffic regime downstream always happens via a capacity regime. The outflow from the tailback is always optimal.

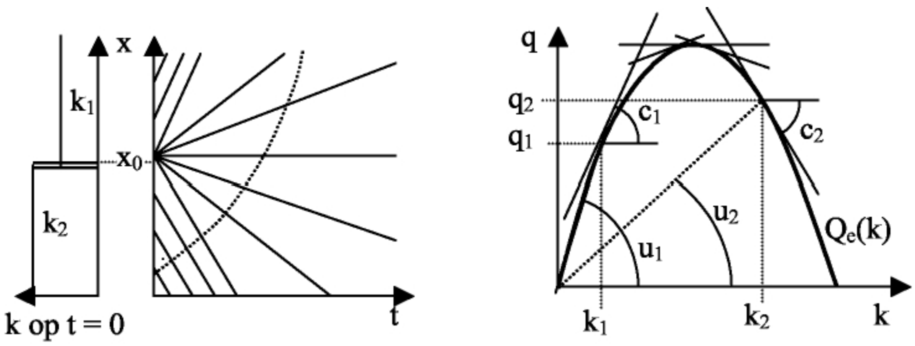

Figure 23 A fan of characteristics at a decrease of density

Example:

We examine a road with traffic state 'A' as initial and boundary condition, except between x_0 en x_1 where for t = 0 to traffic state 'B' applies as illustrated in Figure 24.

A shock wave arises at the transition from 'A' to 'B' and a fan arises at the transition from 'B' to 'A'. The shock wave will continue to be a straight line as long as it keeps the homogeneous states 'A' and 'B' separated. When the characteristics in the fan collide with traffic state 'A' the shock wave becomes a curved line. Note the emergence of the capacity regime at x_1 (a horizontal characteristic). At $x_{1,}$ the slope of the shock wave that makes the transition to traffic state 'A' equals the slope between 'A' and 'C' in the fundamental diagram.

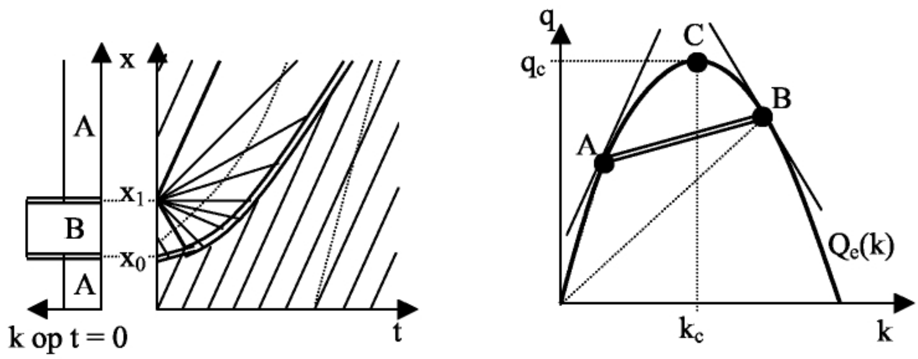

Figure 24 Example using fans and shock waves

With the use of characteristics, shock waves and fans, we can construct a solution starting from the initial and boundary conditions. However, there are additional rules for the boundary conditions. Characteristics with negative speeds, for example, are unable to cross the upstream boundary (the t-axis). Because from that moment there is congestion upstream of our boundary condition and the solution affects the boundary condition itself.

4.5 Triangular fundamental diagram.

To this point we worked with a general concave fundamental diagram. In our further elaboration of the LWR model we will use a triangular fundamental diagram as proposed in chapter 2, Figure 12. The derivative to this diagram is discontinuous. For densities below the capacity density, $Q_e'(k)$ equals the free flow speed u_f. For densities above k_c, $Q_e'(k)$ equals w. We accommodate the discontinuity of $Q_e'(k)$ by assuming that all intermediate values (between u_f and w) occur in k_c.

Applying the triangular fundamental diagram has the following advantages:

- During the free flow regime the speed of the characteristics is u_f. This speed is equal to that of the speed of the vehicles. During free flow, trajectories and characteristics run parallel.
- Shock waves between two states within the free flow regime also have the u_f speed. These shock waves, that run parallel to the characteristics and trajectories, are called ‘slips’.
- Shock waves between two states of congestion run via a fixed speed w.
- The speed of characteristics in fans varies between w and u_f. The density of all these intermediate characteristic speeds is k_c. Thus, the traffic state in a fan is automatically that of the capacity regime.

All of these considerations are illustrated in the following example:

Example:

Again we look at a road with a traffic state 'A' as initial- and boundary constraint, except for the area between x_1 and x_2 where the traffic state 'B' applies in the beginning, as in Figure 25. Traffic state 'C' occurs in the fan between 'B' and 'A'. Here the road functions in the capacity regime. Now the shock wave between traffic state 'A' and the fan is no longer a curve line, but a 'slip': a shock wave that runs parallel to the characteristics and that has a speed of u_f..

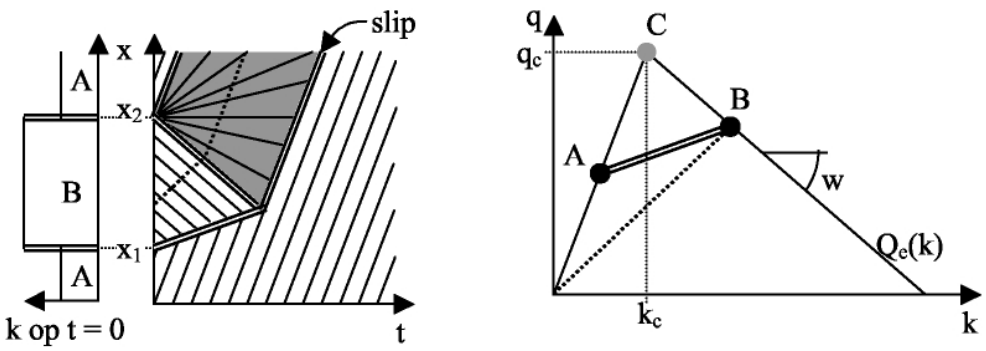

Figure 25 Shock waves and fans with a triangular fundamental diagram

4.6 Non-homogeneous roads

So far we looked at traffic on homogeneous roads. We can now compute the propagation of characteristics, shock waves and fans on such homogeneous road sections. However, disturbances such as shock waves and fans are caused by non-homogeneous points in traffic networks. The conservation of vehicles still applies to these transitions. We will use a number of examples to examine the mechanisms and traffic states.

4.6.1 A traffic light

Our first example examines a traffic light. (see Figure 26). Consider a road where traffic state 'A' functions as initial- and boundary constraint. The traffic light at location x_s, switches to red between t_s and t_e. Traffic directly upstream of the halt line will become completely saturated and cause traffic state 'J'. The flow rate in this state is zero, complying with the stop condition. This causes a shock wave between traffic states 'A' and 'J'. The halt line functions as an upstream boundary constraint with a traffic state 'J' and from this point characteristics leave at speed w against the direction of the traffic stream. The larger the flow rate of traffic state 'A', the larger the speed with which the shock wave propagates against the travel direction.

The traffic downstream of the halt line is in a 'full free flow' state 'O'. The flow rate here is also zero. The shock wave between traffic states 'A' and 'O' is a slip with speed u_f. Here the halt line functions as a downstream boundary constraint pertaining to traffic state 'O' where characteristics leave with speed u_f.

When the stop condition ends at t_e, we can see the road once again as a road with the following initial conditions:

- Traffic state A for $x < x_1$ $(= x_s + (t_e\text{-}t_s) / U_{AJ})$
- Traffic state J for $x_1 < x < x_s$
- Traffic state O for $x > x_s$

The solution to this problem results in a fan between traffic states 'J' and 'O', and two shock waves that will eventually merge in a slip at (t_m, x_m).

The example clearly shows that ending the stop condition leads to a capacity regime on the road and also shows that the length of the queue decreases from that point onwards. When the queue has been dissolved the original traffic state reappears. Also note the abrupt speed changes in the shock waves. In practice deceleration and acceleration will take some time which means that the shock wave will be somewhat elongated.

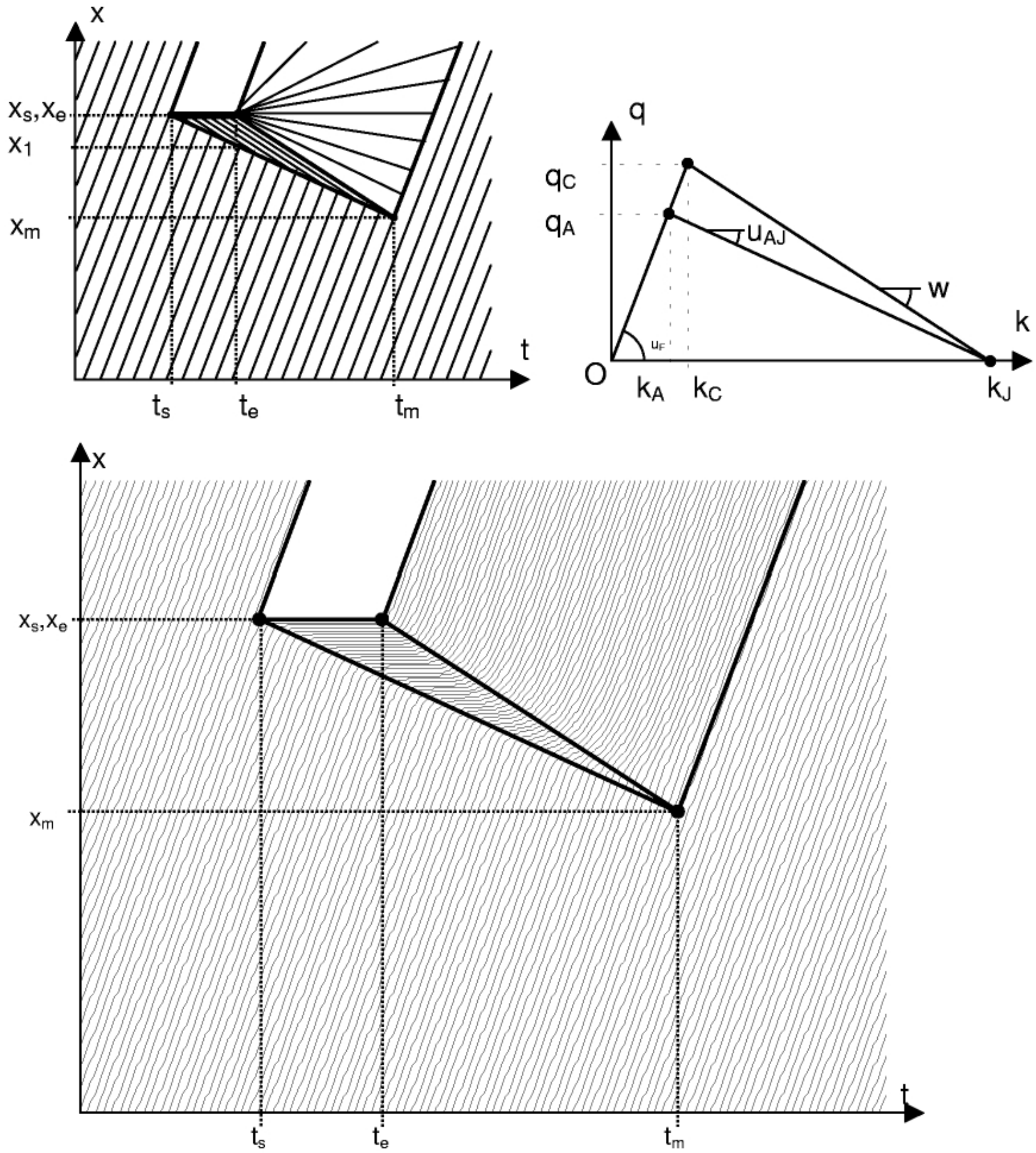

Figure 26 Traffic light (a) characteristics (b) fundamental diagram (c) simulated trajectories

4.6.2 Narrowing of a road with a temporary traffic overload.

In a second example we examine a three-lane road that is reduced to two lanes between x_3 and x_5 (see Figure 28). The maximum speed at the constriction remains at u_f, capacity flow rate and saturated density reduce to two thirds of the original values. We examine the evolution of traffic over this road taking traffic state 'A' as the initial condition. Between t_0 and t_1 traffic state 'D' is the upstream boundary condition and beyond t_1 ‘A’ applies again.

Note that the flow rate q_D exceeds the capacity q_{C2} of the constriction.

The characteristics departing from the initial condition travel at speed u_f. The transition at x_3 and x_5 in not problematic. States 'A' and 'D' are separated from each other by a slip through the origin. This shock wave is able to continue to the constriction without any problems. The constriction can only process a capacity of q_{c2} and this creates a boundary condition downstream: the flow rate will equal capacity q_{c2} of the constriction, and the traffic state will be one of congestion. The flow rate of traffic state 'B' will equal the capacity of the constriction and is located in the congestion area of the fundamental diagram of the three-lane road.

The shock wave between traffic states 'B' and 'D' runs against the flow direction (Q_D exceeds Q_B). At t_1 a shock wave emerges between 'D' and 'A'. When this shock wave meets the one between 'D' and 'B', the traffic regime 'D' disappears permanently. From this point a new forward shock wave begins between 'A' and 'B' and this shock wave reduces the congestion. When this wave reaches the constriction, the tailback has resolved itself. In the constricted section the capacity regime will fan out. At the end of the constriction, at x_5, the continued flow rate causes a continuation of state 'C2'. In the fundamental diagram of the three-lane road, traffic state 'C2' is part of the free traffic regime and its characteristics travel at speed u_f.

We examine the evolution of the traffic states exactly as an observer would do if he was standing by the side of the road. At a sufficient distance upstream from the constriction, in x_1 for example, the observer notes a temporary increase in flow rate q_D. There is no sign of congestion. Closer to the constriction at x_2, our observer would note a temporary increase in flow rate q_D after traffic state 'A' that exceeds the capacity in the constricted section. This is followed by a tailback where the flow rate is equal to the capacity of the bottleneck. The tailback is followed by a free traffic regime with a flow rate below that of the capacity of the constriction.

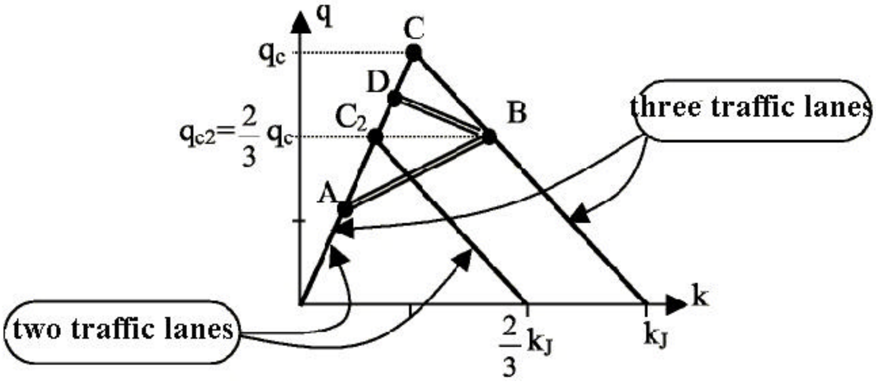

Figure 27 The fundamental k-q diagram for a constricted motorway

Congestion never occurs in a constriction, such as at x_4. The traffic state evolves from 'A' to the capacity regime 'C2' .

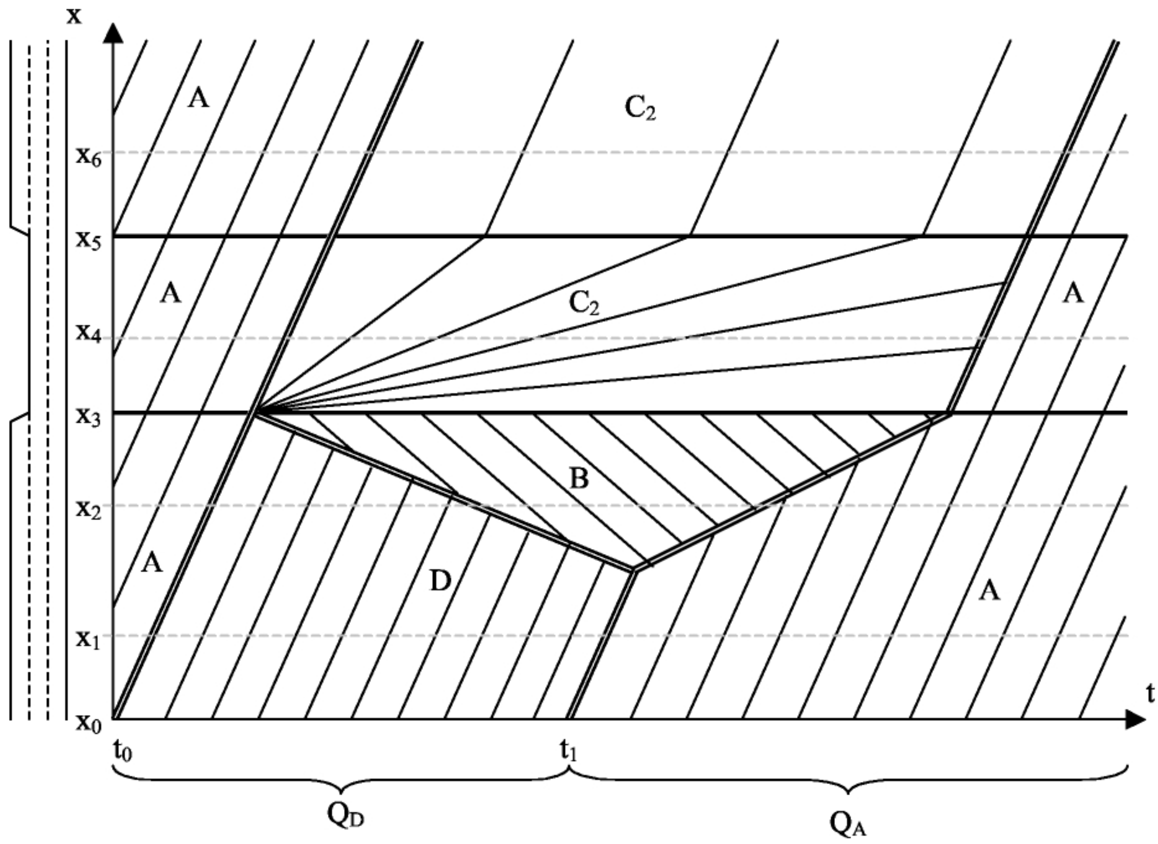

Figure 28 The *t-x* diagram of a motorway with a constricted section

Beyond the constriction, as in x_6, one never sees flow rates that exceed the capacity of the constriction.

An observer **upstream** of a constriction, or more usual of a bottleneck, will only see a temporary flow rate greater than the bottleneck capacity. This higher flow rate will be followed by congestion depending on the distance to the bottleneck

An observer **downstream** from a bottleneck will never see a flow rate that exceeds the capacity of the bottleneck.

The operation of 'bottlenecks' is an important mechanism in the functioning of our road network. Their location and timing determine the location and length of tailbacks. In the example, the physical constriction of the motorway caused a bottleneck. There are other causes for similar 'bottleneck'-effects:

- When a large volume of traffic enters a motorway via a feeder road, the traffic demand beyond the feeder road is significantly larger than the demand upstream of the feeder road. Capacity just beyond the feeder road is reached faster often initiating a bottleneck.

- A local non-homogenous situation (i.e. a number of trucks trailing each other, …) can temporarily reduce local capacity somewhat, which can lead to the formation of a bottleneck.
- Accidents also cause a temporary and local decrease in capacity, which causes the feared 'bottleneck' effect.
- Bad weather reduces capacity. This can be of a very local nature (i.e. the formation of ice on bridges).

5 Microscopic traffic flow models.

In this section traffic is not modelled using aggregate variables such as density, flow rate or mean speed. The microscopic level deals with the interactions between individual drivers, vehicles and the infrastructure.

5.1 General structure

Microscopic traffic models describe the interactions between the various vehicles. Since it is impossible to predict the behaviour of each driver with absolute certainty, stochastic models are commonly used for this purpose. They are implemented in the form of a computer simulation model. Driver- and vehicle properties at time $t + Dt$ are computed on the basis of their values at time t. This is how, for example, the position and speed of all vehicles are computed. In contrast to macroscopic models this method makes it easier to specify different types of vehicles and drivers. The required computing power and the large number of parameters sometimes impede the use of these models.

Most micro-simulation models contain the following components:

- *The car-following model*
  This model assesses the behaviour of a specific vehicle on the basis of the driving behaviour of the vehicle ahead.
- *The lane-change model*
  This deals with the way in which vehicles change lanes based on the vehicles around them.
- *Route choice model*
  As we saw in the static traffic demand model, vehicles need to find the shortest route through an infrastructure network. We use a *dynamic* OD (Origin-Destination) table. The OD table is specified per time period (for 15 minutes, for example).
- *Additional modules*
  Because position, speed and acceleration of each separate vehicle is known at every time segment (for example at time intervals of half a second), it is quite easy to calculate derived effects such as pollution, noise pollution, time loss and economic costs.

In addition to vehicle characteristics, we can also model a number of dynamic characteristics connected to the infrastructure such as traffic lights, weather and accidents.

5.2 Car-following model

In this paragraph we discuss a simple example of the car-following model. This model describes the acceleration of a vehicle using the properties of the car in front of it.

Formula 5.1 assumes that the acceleration is proportional to the speed difference with the car in front. When both vehicles travel at the same speed, the acceleration is zero. It is assumed that the acceleration of a vehicle is inversely proportional to the square of the distance to the vehicle in front. The influence of the vehicle in front increases as the distance between the two vehicles decreases.

$$a_\alpha(t+T_r) = Sens * \frac{\Delta v_\alpha(t)}{s_\alpha(t)^2} \qquad \textbf{(5.1)}$$

This formula has two parameters:

- T_r : The reaction time of the vehicle. Either the driver's reaction to changes is delayed, or he reacts to changes that happened some time T_r ago.
- *Sens*: Driver sensitivity. This factor models the intensity of the reaction of a driver to changes in the behaviour of the car in front of him.

The figures below show the following behaviour of a vehicle. Both vehicles depart from a stationary position. The second driver follows at a distance (or rather a space) of 100 meters while the first car accelerates during 20 seconds at a rate of 1 m/s² and then brakes to stand-still at a rate of –1 m/s². The reaction time amounts to 1 second and sensitivity is 5000 m²/s.

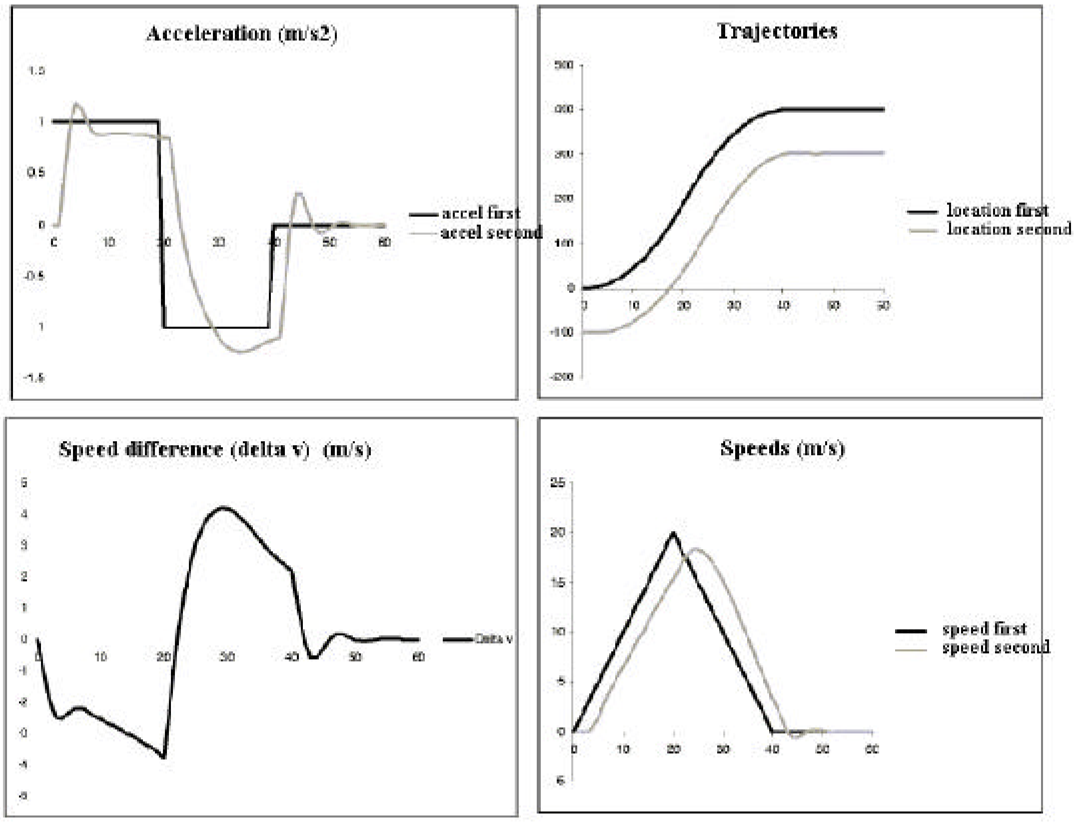

Figure 29 An experiment using the car-following model formula (5.1)

In stationary and homogeneous traffic the speed difference and consequently the acceleration is always zero. In such circumstances a link can be made to the fundamental diagrams by integrating both sides of formula (5.1) with respect to time:
(keeping in mind that: $ds_a(t) / dt = Dv_a(t)$)

$$v_\alpha(t+T_r) = \frac{-Sens}{s_\alpha(t)} + C \qquad (5.2)$$

In this expression C is an integration constant. In homogeneous and stationary traffic the speed is constant and the same for all vehicles. This means that the reaction time T_r is of no importance and the mean speed u equals v_a. The space occupied is equal for all vehicles and consequently equals the average space *s.* We can apply the link with density from 2.2 to arrive at the following:

$$u = -Sens..k + C \qquad (5.3)$$

The integration constant and the sensitivity level can then be taken from the boundary conditions:

- at a density of zero, the speed is u_f.
- at a speed of zero, the density is at its maximum and equals k_j

This leads to the expression:

$$u = -\frac{u_f}{k_j} k + u_f \qquad (5.4)$$

Expression 5.4 describes the relation from the fundamental *k-u* diagram and it corresponds to the formulation (3.1). This car-following model was constructed in such a way that for stationary and homogeneous traffic it leads to the fundamental diagrams of Greenshield. Different car-following models, in their turn, lead to different fundamental diagrams.

6 A real-life tailback

This last chapter analyses an actual traffic pattern and discusses some additional effects.

6.1 Description of the road section

As our example we will use an eight-kilometre long section of the E17 Gent - Antwerp motorway just before the Kennedy tunnel. The motorway has three feeder- and exit roads on the right followed by an exit road on the left and two feeder roads on the left-hand side. The motorway bends to the right between kilometres 6 and 7. Fifteen camera detectors, numbered from CLO F to CLO I, measure the traffic flow rate (vehicles/min) and the average speed (km/hour) per minute on the three traffic lanes. The study area is shown schematically in Figure 30.

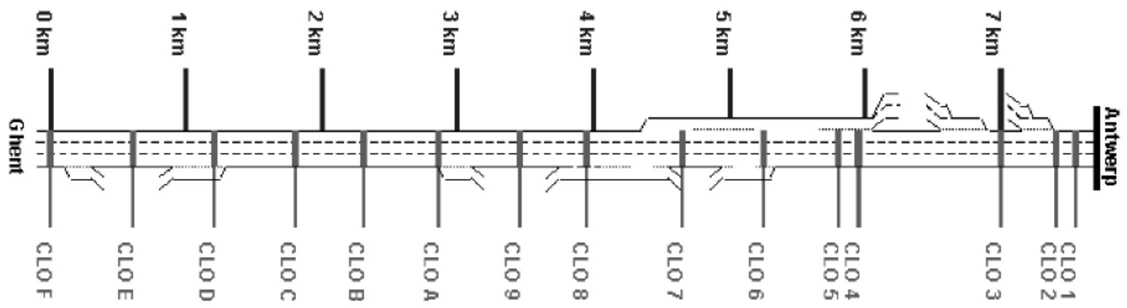

Figure 30 A part of the E17 with traffic detectors

Figure 31 shows the flow rate and average speeds of the three traffic lanes on 28 September 1999. The time axis is drawn horizontally, the location vertically. Vehicles travel from bottom left to top right.

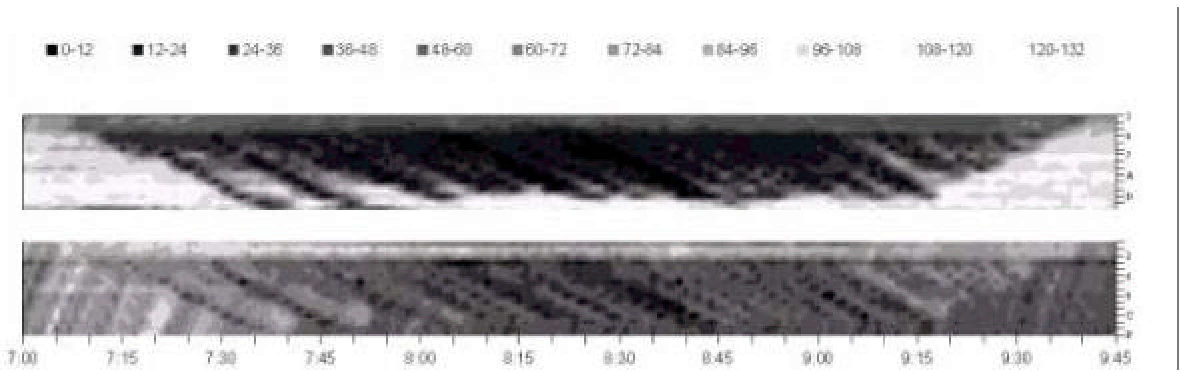

Figure 31 Observations : (above) average speed [km/hour],(below) flow rate [veh/min]

6.2 Analysis according to the macroscopic traffic flow model..

First the measurement results will be discussed with the macroscopic traffic flow model of chapter 4 in mind. This traffic model distinguishes three traffic regimes: 'Free flow', 'capacity flow and 'congested flow'.

Before 7.10 am traffic moves in the 'free flow regime'. In this regime vehicles travel at high speed and the state points lie on the top branch of the fundamental flow-speed diagram. The traffic state depends on the state upstream of the road section under examinatio n. This is why small fluctuations in the traffic demand give rise to waves that move in the travel direction. These waves can only be seen when one looks at the flow rate. This indicates that the speed is independent of the flow rate. In other words, we are dealing with a horizontal branch in the flow-speed diagram (as in Figure 12 on the right).

At the CLO3 detector the capacity of the road is reached at 7.10 am, due to an increased supply via the first left feeder road. This 'capacity regime' continues to 9.30 am and the state points of this regime lie on the extreme right in the fundamental flow-speed diagram.

The 'congestion regime' originates in the bottleneck and propagates against the travel direction. Speed is low and the traffic state is determined by the bottleneck upstream. We now are on the lower branch in the fundamental flow-speed diagram.

The three regimes that are derived from the macroscopic traffic flow model, are clearly distinguishable in the traffic data.

6.3 *Additional empirical characteristics*.

The traffic measurements show that there are effects other than the three regimes. These additional effects cannot be explained on the basis of the traffic flow model.

The flow leaving the bottleneck lies below the maximum flow rate that is achieved during the 'free-flow regime'. This is why the flow rate of the traffic downstream of the bottleneck, a 'free flow' regime with the bottleneck as boundary condition, lies below that of the bottleneck regime. We call this effect the *capacity drop*.

It also appears that the bottleneck starts to operate when the flow rate exceeds 100 vehicles per minute across the three traffic lanes and that it disappears only when the flow rate goes below 70 vehicles per minute. This *hysteresis effect*, therefore, prolongs the bottleneck regime beyond what would appear necessary. Figure 32 plots the various state points at the bottleneck location in a *q-u* diagram. When these points are chronologically connected it appears that the start and the end of bottleneck regimes follow different paths.

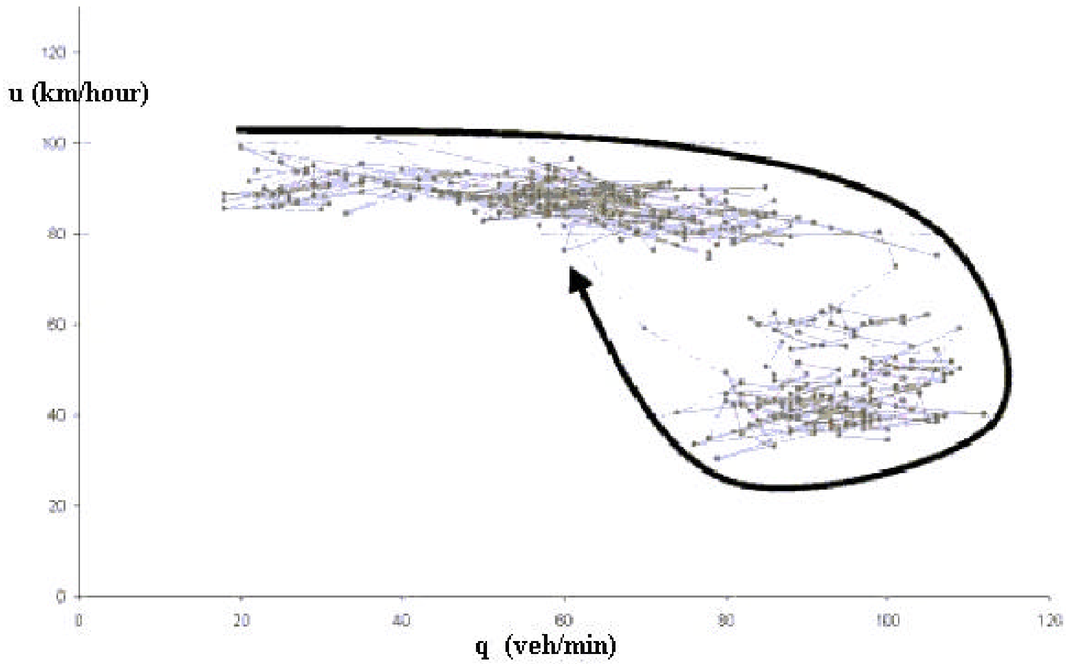

Figure 32 Fundamental q – u diagram where successive state points have been connected

The waves inside the 'congestion regime' in Figure 32 cannot be explained using the macroscopic model in chapter 4. These *start-and-stop-waves* emerge through small disturbances in the bottleneck and they develop into larger waves of strongly varying flow rate and speed.

The first two congestion waves have a period of 10 minutes. There is, in fact, a period of 'free flow' traffic in between these waves. Subsequent waves show lower vehicle speeds and there is progressively less time between each wave. Characteristic of these waves is that they move at constant speed against the travel direction. Drivers cross these waves and experience them as a succession of acceleration and deceleration.

List of figures